



# Statistical Design and Planning of an Adaptive Trial w/ Hierarchical Composite Outcomes: A Practical Example

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# Summary

- Clinical trials using composite outcomes in a hierarchy are gaining popularity, especially in the CV space.
- The Finkelstein and Schoenfeld (FS) test is a generalized pairwise comparison approach to analyze prioritized composite endpoints
- Using the FS-method as the primary statistical analysis and quantifying the magnitude of effect using Win ratio (WR) is a strategy many sponsors are employing in this situation.
- However, adaptive trial designs using this analytic approach are not as well-known.
- This presentation describes the successful design and implementation of an adaptive clinical trial with composite hierarchical endpoints, using an SSR at an Interim Analysis.

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# Finkelstein and Schoenfeld (FS) Method

- Used frequently in cardiovascular trials that have hierarchical composite outcomes.
- Pairwise comparisons based on a clearly pre-specified clinical hierarchy:
  - Mortality (highest priority)
  - Cardiovascular hospitalizations (CVH)
  - Functional improvement (6MWT, NT-proBNP, PRO instrument etc.)
- Method is based on pairwise comparison of all patients in the trial across each of the 3 endpoints in order to determine a non-parametric estimate of the number of “wins”.

# FS-Method

## Steps for the Finkelstein–Schoenfeld Test

### 1. Create Pairwise Comparisons:

- ▶ Randomize  $n_1$  patients to placebo and  $n_2$  to active treatment.
- ▶ Form all  $n_1 \times n_2$  pairwise comparisons (patient  $i$  in placebo vs. patient  $j$  in active).

### 2. Compare the Highest-Priority Endpoint (Mortality):

- ▶ If one patient died and the other did not, the group without death gets a “win.”
- ▶ If both died or both survived, proceed to the next endpoint.

### 3. Compare of Cardiovascular Hospitalizations:

- ▶ If one patient had more CV hospitalizations than the other in the shorter of the follow up times, then they “win”
- ▶ If tie cannot be broken, proceed to the next endpoint.

### 4. Compare Functional Improvement:

- ▶ If one patient’s 6MWT improvement is greater, assign a “win” to that treatment arm.
- ▶ If both are equal, this pair is considered a “tie.”
- ▶ Sum up all wins/losses (+1 or -1) across all pairs to obtain the overall Finkelstein–Schoenfeld statistic.

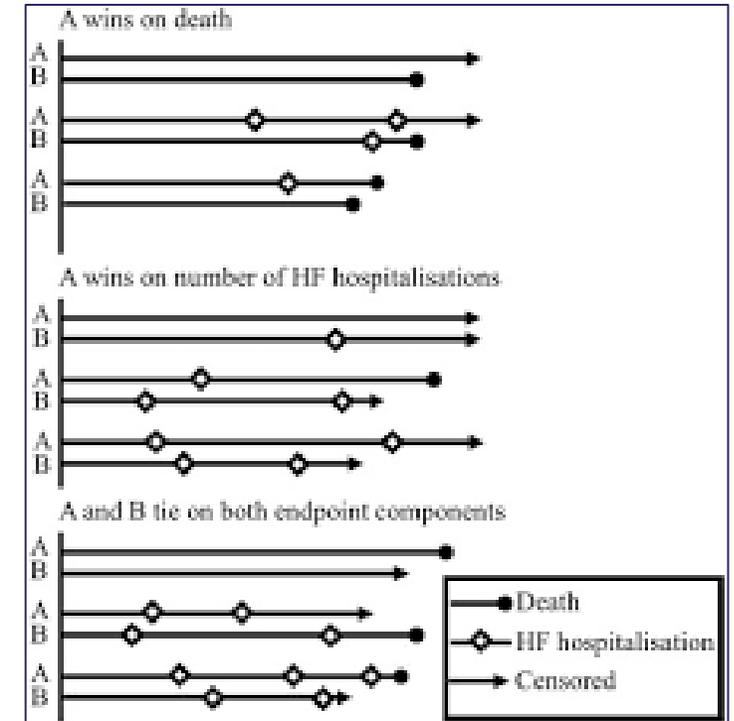


Image from - Redfors et. Al: *European Heart Journal* (2020) 41, 4391–4399

# Large Sample distribution

▶ Test statistic:

$$T = \sum_{i=1}^{2n} Z_i U_i$$

▶ Mean:

$$E(T) = 2n^2(\theta - 1/2)$$

▶ Variance:

$$\text{Var}(T) = \frac{n^2}{2n(2n-1)} \sum_{i=1}^{2n} U_i^2$$

- $U_i$  is the score obtained by summing all the wins (+1) and losses (-1) for an individual patient  $i$ .
- $T$  is the test statistic calculated by summing the total wins and losses across all patients for the active treatment.
- $\theta$  is the probability that a random subject  $i$  in the treatment group has a better outcome than a random subject  $j$  in the control group.
- Therefore,  $H_0$  is:  $\theta = 1/2$

# Win Ratio: Quantifying Treatment Benefit

## 1. Count Wins & Losses

- ▶ Record  $Wins_{active}$  and  $Losses_{active}$
- ▶ Unresolved pairs  $\rightarrow$  *Ties* (ignored)

## 2. Compute the Win Ratio

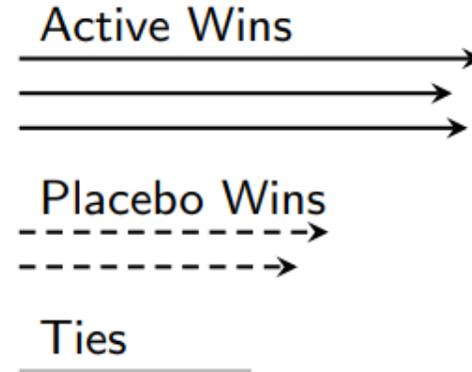
$$WR = \frac{Wins_{active}}{Losses_{active}}$$

## 3. Confidence Interval & p-Value

- ▶ Delta-method or bootstrap variance on pair scores
- ▶ 95 % CI (log-scale back-transformed)

## 4. Clinical Interpretation

- ▶  $WR = 1.40 \rightarrow$  "A 40% higher likelihood of a better outcome with the treatment compared to control"



Example: Wins = 180, Losses = 128, Ties = 62  
 $WR = 180/128 = 1.40$

# Trial Design for new CV investigational agent

- Hierarchical Endpoints:
    - Mortality
    - # of CV Hospitalizations
    - Functional response (0/1)
  - Possible Adaptations at IA
    - Non-binding futility
    - SSR
- ▶ Sample size:  $N = 400$ , interim at  $N = 200$ .
  - ▶ Historical rates:
    - ▶ Mortality: 30% (Active) vs. 40% (Control).
    - ▶ CVH Hazard: 0.25 (Active) vs. 0.4 (Control).
    - ▶ Functional response: 50% (Active) vs. 25% (Control).
  - ▶ Predictive Probability (PP) determines SSR decision.
  - ▶ Decision zones:
    - ▶  $PP < 10\%$ : Stop for futility.
    - ▶  $10\% \leq PP < 30\%$  or  $PP \geq 90\%$ : Continue with  $N = 400$ .
    - ▶  $30\% \leq PP < 75\%$ : Increase to  $N = 600$ .
    - ▶  $75\% \leq PP < 90\%$ : Increase to  $N = 500$ .

# Optional Initial Sample Size Determination: Using Win Ratio An Alternative to Simulation based approach

(via Yu, Ganju 2021)

## Win Ratio

- The win ratio (WR) extends the FS statistic by expressing the results as a ratio.
- The win ratio = (number of favorable comparisons) / (number of unfavorable comparisons).
- It summarizes how often the treatment group "wins" (has a better outcome) compared to the control.
- A win ratio >1 suggests that the treatment is beneficial.

- Simulate patient data per the desired HA.
- Estimate empirical WR
- Estimate p\_tie
- Use asymptotic distribution to calculate N\_Init.
- N\_Max can be varied based on WR under MCID.

The approximate variance of the log of the win ratio is

$$\text{Var}(\ln(WR)) \approx \frac{1}{N} \times \left\{ \frac{4(1 + p_{\text{tie}})}{3k(1 - k)(1 - p_{\text{tie}})} \right\} \quad (1)$$

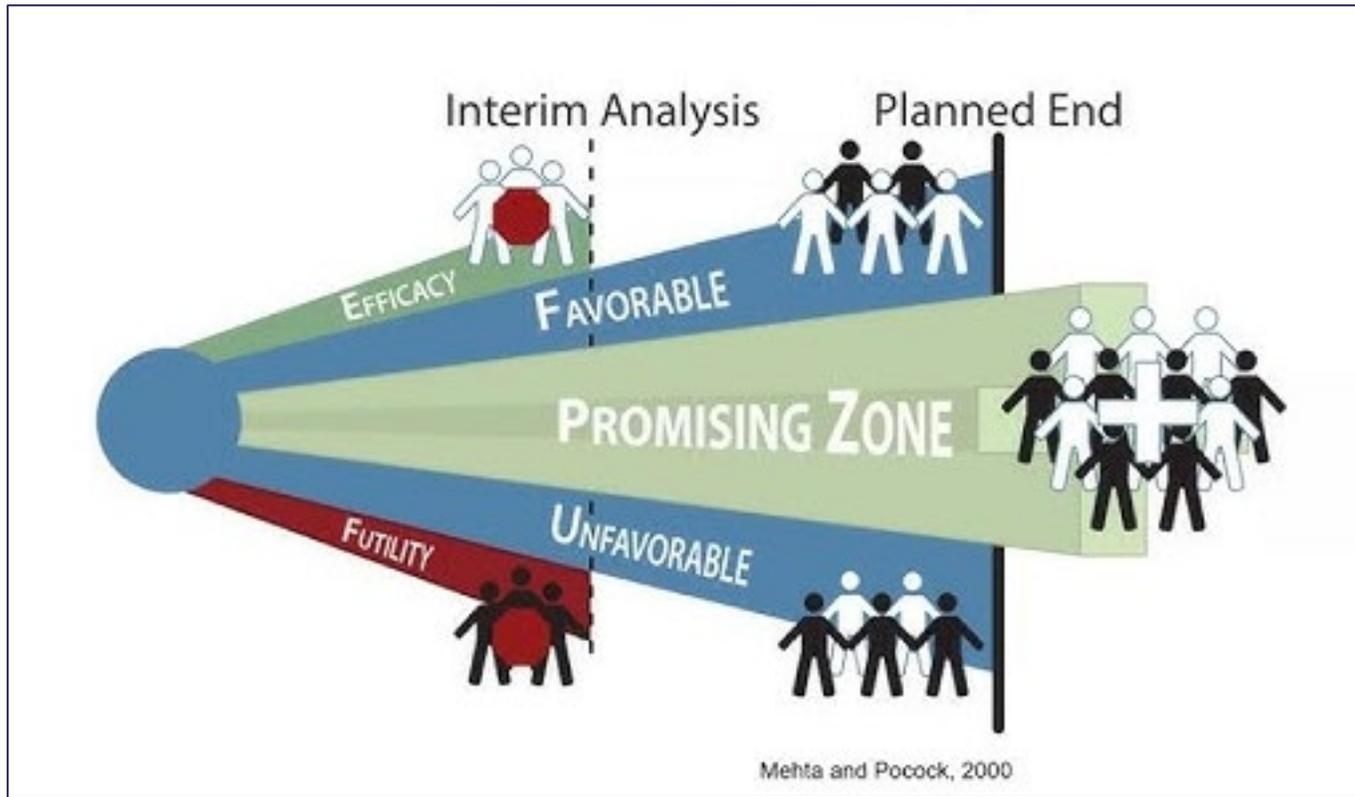
Using conventional notation, we write this as  $\frac{1}{N} \times \sigma^2$ , where  $\sigma^2$  represents the term within {}.

The required sample size is

$$N \approx \frac{\sigma^2 \times (Z_{1-\alpha} + Z_{1-\beta})^2}{\ln^2(WR_{\text{true}})} \quad (2)$$

where  $WR_{\text{true}}$  refers to the assumed or true value of the win ratio, and  $\ln(WR_{\text{true}})$  refers to its natural log-transformed value. As usual,  $\alpha$  and  $\beta$  refer, respectively, to type I and II error rates

# Adaptive Design with unblinded SSR



## Operating Principles:

- Ability to increase SS at a pre-specified IA to re-size the study appropriately
- Typically operationalized using conditional Power
- Pre-specified IA timing, Max SS, T1E methodology and Final analysis method are required
- Frequently accepted by agencies as a well-understood adaptive design.

# CP vs Approx. PP: Both result in similar OCs

CP provides finer control, but approx. PP is easier to implement

The first 200 participants (from both arms combined) will be assigned to cohort 1 while the subsequent 200 planned participants (from both arms combined) will form cohort 2  
Let

$$w_i = \sqrt{\frac{n_i}{n}}, \quad i = 1, 2,$$

be the weights that we will use to combine the F-S statistic  $T_1$ , obtained from the cohort 1 data, and the F-S statistic  $T_2$ , obtained from the cohort 2 data, such that the standardized test statistic for testing the null hypothesis at the final analysis is

$$w_1 \frac{T_1}{s_1} + w_2 \frac{T_2}{s_2},$$

where  $s_j$  is the standard error of  $T_j$ . Then the CP, given  $T_1 = t_1$ , is

$$CP(t_1, n_2) = P \left( w_1 \frac{t_1}{s_1} + w_2 \frac{T_2}{s_2} > z_\alpha \mid T_1 = t_1 \right).$$

Since  $T_1, T_2$  are independent and asymptotically normal, the CP can be evaluated as

$$CP(t_1, n_2) = 1 - \Phi \left( z_\alpha - \frac{w_1 t_1}{s_1} - w_2 \frac{E(T_2/s_2)}{\sqrt{n_2}} \right),$$

where  $E(T_2/s_2)$  is estimated by repeatedly generating simulated responses for the  $n_2$  subjects in cohort 2.

$$PP(p_n, r, \alpha) =$$

$$\Phi \left( \frac{\Phi^{-1}(1 - p_n) - \Phi^{-1}(1 - \alpha) \sqrt{r}}{\sqrt{1 - r}} \right)$$

Image from – Marion et. al. 2024

# Final analysis

- Predictive Power & Sample Size Re-evaluation: After estimating predictive power, stage 2 data is collected, and the final analysis occurs once all subjects complete 12 mo. or are terminated early.
- Hypothesis Testing: The null hypothesis is rejected based on the weighted combination of test statistics from both stages.
- Rejection Criteria:

- No SSR at IA:  $w_1 \frac{t_1}{s_1} + w_2 \frac{t_2}{s_2} \geq z_\alpha$

Equivalent to CHW method, that controls Type I error.

- SSR at IA:  $w_1 \frac{t_1}{s_1} + w_2 \frac{t_2^*}{s_2^*} \geq z_\alpha$

Weights  $w_1 = \sqrt{n_1/n}$  and  $w_2 = \sqrt{n_2/n}$  remain fixed

# Simulation Study: Joint Frailty Model and Scenarios

For person  $i$ , let  $T_{i0} = 0$  and  $T_{i1}, T_{i2}, \dots, T_{iN_i}$  represent recurrent event times. Here,  $N_i$  counts events before  $X_i = \min(C_i, D_i)$ , where  $C_i$  is censoring time and  $D_i$  is CV death time. The hazard functions define the JFM:

$$r_i(t | \omega_i) = \omega_i \exp\{\beta_1 z_i\} r_0(t) = \omega_i r_i^*(t)$$
$$\lambda_i(t | \omega_i) = \omega_i^\alpha \exp\{\beta_2 z_i\} \lambda_0(t) = \omega_i^\alpha \lambda_i^*(t).$$

For patient  $i$ ,  $r_i$  gives the hospitalization hazard, scaling with baseline intensity  $r_0$  and frailty  $\omega_i$ . The CV death hazard  $\lambda_i$  builds on baseline  $\lambda_0$ , with  $\beta_1, \beta_2$  capturing treatment effects ( $z_i$ ). Random effects  $\omega_i$  follow a lognormal distribution (mean 0, variance  $\theta$ ).

Table 2: Hypothetical Scenarios to be Explored

Five Hypothetical Scenarios (Active vs. Control)					
Event Rates	Null	Alternative	Alternative2	Middling	Middling2
Mortality	40% vs 40%	30% vs 40%	40% vs 40%	32.5% vs 40%	35% vs 40%
CVH Rate	0.4 vs 0.4	0.25 vs 0.375	0.10 vs 0.40	0.275 vs 0.40	0.28 vs 0.4
Func. Resp.	25% vs 25%	50% vs 25%	60% vs 25%	45% vs 30%	40% vs 25%

# Results (10K runs for each scenario)

Table 3: Trial Operating Characteristics Without Sample Size Re-estimation

Scenario	Power (%)	Average N	Futility Stop (%)
Null	2.3	261.5	68.2
Alternative	90.9	378.6	2.8
Alternative2	73.4	384.3	7.9
Middling	63.6	377.0	11.4
Middling2	47.0	348.6	19.2

Table 4: Trial Operating Characteristics With Sample Size Re-estimation

Scenario	Power (%)	Average N	Futility Stop (%)	Max SSR (%)
Null	1.9	286.8	69.1	12.5
Alternative	94.5	437.0	2.5	21.3
Alternative2	81.5	473.4	8.4	45.1
Middling	72.2	471.7	11.7	48.1
Middling2	53.3	425.6	20.0	32.8

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# Conclusions

## Fixed Design Without SSR:

- Strong Type I error control (2.3%) with high futility stopping under the null (68.2%).
- High power (82.9% - 90.9%) in alternative scenarios with moderate sample sizes (373 - 379).
- Power drops significantly in middling scenarios (47 - 52%), highlighting risk when true effects are marginal.

## Adaptive Design with SSR:

- Maintains Type I error control (1.9%) while preserving high futility stopping (69.1%).
- Increases power in alternative scenarios (88.7-94.5%) at the cost of larger avg. N (437 - 444).
- Middling scenarios show modest power improvements (53 - 59%), but SSR often requires maximum sample size (600 patients in 33% of trials).
- Suggests SSR helps address power deficits for smaller treatment effects but may demand substantial increases in sample size.

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# Key Takeaways

- **Benefits of Adaptive SSR:** Improves trial efficiency, cost savings, flexibility, and statistical validity.
- **Application Challenges:** Common for simple endpoints (continuous, binary, survival) but less used for complex endpoints.
- **Proposed Approach:** Uses an Adaptive design with SSR for a CV trial with F-S statistic with multiple hierarchical endpoints.
- **Clinical Impact:** Enhances adaptive design applicability, particularly in cardiovascular trials using the F-S statistic.
- **Real World Application:** This is a case study inspired by (but does not show numbers and/or the exact design) a CV project on which the authors consulted with a sponsor on. Quantification of treatment effect is a simple extension using Win Ratio.
  - The trial read out in 2023-24 and was positive for the final analysis.
  - IA SSR decision was: No change to SS, since z-statistic was in the favorable zone.

# Full Paper on arXiv

The screenshot shows the arXiv interface for a paper. At the top left is the Cornell University logo. A search bar is located at the top right. The breadcrumb trail is 'arXiv > stat > arXiv:2504.14748'. The paper title is 'Statistical Design and Planning of an Adaptive Trial using Hierarchical Composite Outcomes: A Practical example', submitted on 20 Apr 2025, by Krishna Padmanabhan and Cyrus Mehta. The abstract discusses hierarchical composite endpoints and adaptive design methods. On the right, there are links for 'Access PDF', 'HTML (export)', 'TeX Source', and 'Other Formats', along with a Creative Commons license (CC BY-NC-SA) and a 'stat.ME' category link.

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[Submitted on 20 Apr 2025]

## Statistical Design and Planning of an Adaptive Trial using Hierarchical Composite Outcomes: A Practical example

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Hierarchical composite endpoints, such as those analyzed using the Finkelstein-Schoenfeld (FS) statistic, are increasingly used in clinical trials for their ability to incorporate clinically prioritized outcomes. However, adaptive design methods for these endpoints remain underdeveloped. This paper presents a practical framework for implementing sample size re-estimation (SSR) in trials using hierarchical composites, motivated by a cardiovascular trial with mortality, hospitalization, and a functional response as prioritized endpoints. We use a two-stage adaptive design with a single interim analysis for illustration. The interim analysis incorporates predictive probabilities to determine whether the trial should stop for futility, continue as planned, or increase the sample size to maintain power. The decision framework is based on predefined zones for predictive probability, with corresponding adjustments to the stage 2 sample size. Simulation studies across various treatment scenarios demonstrate strong type I error control and increased power compared to a fixed design, particularly for treatment effects that are clinically relevant but lower than the alternative hypothesis. We also explore an alternative conditional power approach for SSR, offering further sample size optimization. Our results support the use of SSR with hierarchical composite outcomes using an FS statistic, enhancing trial efficiency.

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# Key References

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